AP Calculus AB
Substitution Worksheet #2

Name Key

Integrate each of the following.

1. $\int xe^{x^2}dx$

$u = x^2$ $\quad \int x e^u \cdot \frac{du}{2x}$

$\frac{du}{dx} = 2x$ $\quad \frac{1}{2}\int e^u du$

$dx = \frac{du}{2x}$ $\quad \frac{1}{2}e^u + C$

$\frac{1}{2}e^{x^2} + C$

2. $\int t^2 \cos(1-t^3)dt$

$u = 1-t^3$ $\quad \int t^2 \cos u \cdot \frac{du}{-3t^2}$

$\frac{du}{dt} = -3t^2$ $\quad -\frac{1}{3}\int \cos u\, du$

$dt = \frac{du}{-3t^2}$ $\quad -\frac{1}{3}\sin u + C$

$-\frac{1}{3}\sin(1-t^3) + C$

3. $\int x(x^2+1)^{3/2}dx$

$u = x^2+1$ $\quad \int x\, u^{3/2} \cdot \frac{du}{2x}$

$\frac{du}{dx} = 2x$ $\quad \frac{1}{2}\int u^{3/2}\, du$

$dx = \frac{du}{2x}$ $\quad \frac{1}{2}\cdot\frac{2}{5}u^{5/2} + C$

$\frac{1}{5}(x^2+1)^{5/2} + C$

4. $\int \sqrt[5]{3-5y}\, dy$

$u = 3-5y$ $\quad \int u^{1/5} \cdot \frac{du}{-5}$

$\frac{du}{dy} = -5$ $\quad -\frac{1}{5}\int u^{1/5}\, du$

$dy = \frac{du}{-5}$ $\quad -\frac{1}{5}\cdot\frac{5}{6}u^{6/5} + C$

$-\frac{1}{6}(3-5y)^{6/5} + C$

5. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$

$u = x^{1/2}$ $\quad \int \frac{\cos u}{\sqrt{x}} \cdot 2\sqrt{x}\, du$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$ $\quad 2\int \cos u\, du$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ $\quad 2\sin u + C$

$dx = 2\sqrt{x}\, du$ $\quad 2\sin\sqrt{x} + C$

6. $\int \cot x\, dx \rightarrow \int \frac{\cos x}{\sin x}dx$

$u = \sin x$ $\quad \int \frac{\cos x}{u} \cdot \frac{du}{\cos x}$

$\frac{du}{dx} = \cos x$ $\quad \int \frac{1}{u}\, du$

$dx = \frac{du}{\cos x}$ $\quad \ln|u| + C$

$\ln|\sin x| + C$

7. $\int e^x \sin e^x dx$

$u = e^x$ $\quad \int e^x \sin u \cdot \frac{du}{e^x}$

$\frac{du}{dx} = e^x$ $\quad \int \sin u\, du$

$dx = \frac{du}{e^x}$ $\quad -\cos u + C$

$-\cos e^x + C$

8. $\int \frac{\sin x}{1+\cos^2 x}dx$

$u = \cos x$ $\quad \int \frac{\sin x}{1+u^2} \cdot \frac{du}{-\sin x}$

$\frac{du}{dx} = -\sin x$ $\quad -\int \frac{1}{1+u^2}\, du$

$dx = \frac{du}{-\sin x}$ $\quad -\arctan u + C$

$-\arctan(\cos x) + C$